

Question Paper

Physics Multiple Correct (Maximum Marks: 32)

Question No. 1

One or More Options Correct Type

The question has four choices (A), (B), (C) and (D) out of which ONE or MORE are correct.

Consider 3 waves described by following electric fields.

$$E_1 = \frac{E_0}{\sqrt{2}} [\hat{x} \cos(\omega t - kz) + \hat{y} \sin(kz - \omega t)]$$

$$E_2 = E_0 \hat{x} \cos(\omega t - kz + \pi/4) + \frac{E_0}{\sqrt{3}} \hat{y} \cos(\omega t - kz + 5\pi/4)$$

$$E_3 = \frac{E_0}{\sqrt{2}} \hat{x} \cos(\omega t - kz) + \frac{E_0}{\sqrt{3}} \hat{y} \sin(\omega t - kz)$$

Choose the correct options.

- A. A particle of charge q sits on $z = 0$ under field E_1 . At any $t > 0$, the instantaneous speed of q is aE_0/w
- B. Light wave E_2 is not linearly polarized
- C. Light wave E_3 is elliptically polarised in clockwise rotation
- D. Light wave E_3 is elliptically polarised in counter clockwise rotation.

Question No. 2

One or More Options Correct Type

The question has four choices (A), (B), (C) and (D) out of which ONE or MORE are correct.

A solid sphere of mass M is placed on a smooth horizontal surface. A sudden blow is given horizontally to the sphere at a height $h = 4R/5$ above the centre line. If I is the impulse of the blow.

- A. The minimum time after which the highest point will touch the ground is $\frac{MR\pi}{2I}$
- B. The displacement of the centre of mass during this interval is $\frac{R\pi}{2}$
- C. Angular velocity of sphere just after impulse provided is $\frac{I}{MR}$
- D. Velocity of sphere just after impulse provided is $\frac{I}{M}$

Question No. 3

One or More Options Correct Type

The question has four choices (A), (B), (C) and (D) out of which ONE or MORE are correct.

A potentiometer has a cell of emf 10 V and internal resistance 2Ω . It is connected to a resistance box in which we can have any integral resistance from 1Ω to 20Ω . The resistance of the potentiometer wire is 10Ω and its length is 4.000 m. The least count of the scale is 1 mm. We wish to measure the emf of a secondary cell whose emf is 7 V.

- A. The reading will be most accurate when we set the resistance box at 2Ω .
- B. The reading will be most accurate when we set the resistance box at 3Ω .
- C. If the resistance box is set at 1Ω and we balance the secondary cell on the potentiometer wire, the null point will be at 3.640 m.
- D. If the resistance box is set at 8Ω and we balance the secondary cell on the potentiometer wire, the null point will be at 2.000 m.

Question No. 4

One or More Options Correct Type

The question has four choices (A), (B), (C) and (D) out of which ONE or MORE are correct.

A gas containing hydrogen like ions with atomic number z emits photons in transition $n + 2 \rightarrow n$ where $n = z$. These photons fall on a metallic plate and eject electrons having minimum de-Broglie wavelength = 5 Å. Find the value of z . Work function of the metal is 4.2 eV.

- A. $n + 3z = 9$

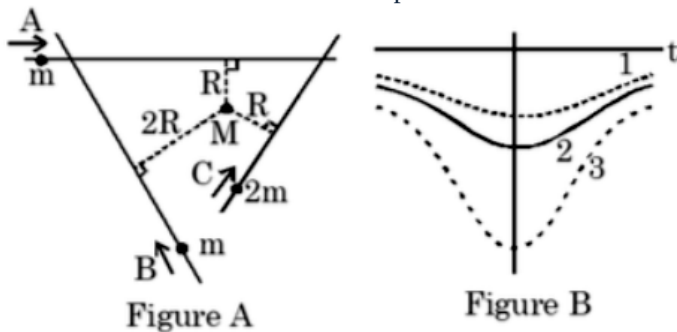
- B. $3n + z = 8$
 C. $8n - 3z = 10$
 D. $3n + 4z = 14$

Question No. 5

One or More Options Correct Type

The question has four choices (A), (B), (C) and (D) out of which ONE or MORE are correct.

In figure A, a stationary spacecraft of mass M is passed by asteroid A of mass m , asteroid B of the same mass m , and asteroid C of mass $2m$. The asteroids move along the indicated straight paths at the same speed; the perpendicular distances between the spacecraft and the paths are given as multiples of R . Figure B gives the gravitational potential energy $U(t)$ of the spacecraft-asteroid system during the passage of each asteroid treating time $t = 0$ as the moment when separation is minimum. Which asteroid corresponds to which plot of $U(t)$?



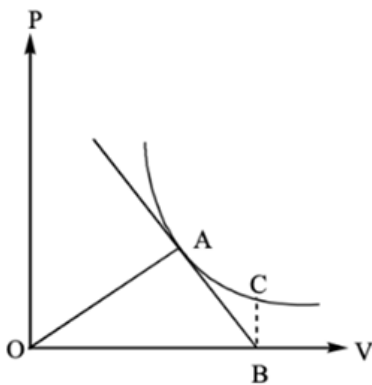
- A. B - 1
 B. A - 2
 C. C - 3
 D. A - 3

Question No. 6

One or More Options Correct Type

The question has four choices (A), (B), (C) and (D) out of which ONE or MORE are correct.

n moles of an ideal gas undergo an isothermal process at temperature T . P-V graph of the process is as shown in the figure. A point $A(V_1, P_1)$ is located on the P-V curve. Tangent at point A, cuts the V-axis at point B. AO is the line joining the point A to the origin O of PV diagram. BC is perpendicular to OB. Then



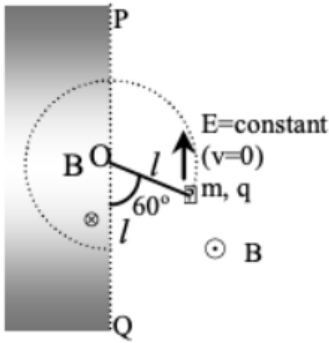
- A. The coordinates of point C are $(2V_1, \frac{P_1}{2})$
 B. The coordinates of point C are $(\frac{3}{2}V_1, \frac{2}{3}P_1)$
 C. Area of the triangle AOB is equal to nRT
 D. Area of the triangle AOB is equal to $\frac{nRT}{2}$

Question No. 7

One or More Options Correct Type

The question has four choices (A), (B), (C) and (D) out of which ONE or MORE are correct.

Consider a bob of mass m and having charge q attached with a light string of length l and pivoted at point O . It is released at rest at 60° with vertical. There are two regions- Region-I (left of line PQ) has a uniform and constant magnetic field B directed inside plane of paper. Region-II (right of line PQ) has a constant and uniform electric field E directed vertically up as shown. Consider no effect of gravity in both the regions.



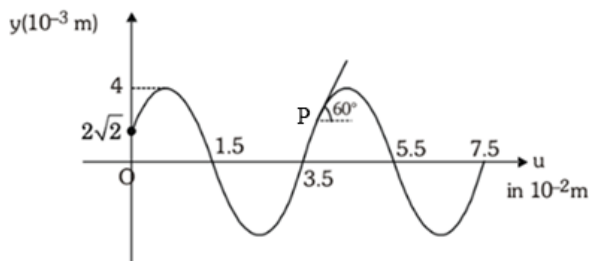
- A. Time taken by particle to cross region-I for 1st time is $\pi\sqrt{\frac{2ml}{5qE}}$
- B. Time taken by particle to cross region-I for 2nd time is $\pi\sqrt{\frac{2ml}{13qE}}$
- C. Angular speed of 1st revolution in magnetic field is $\sqrt{\frac{5qE}{2ml}}$
- D. Angular speed of 2nd revolution in magnetic field is $\sqrt{\frac{13qE}{ml}}$

Question No. 8

One or More Options Correct Type

The question has four choices (A), (B), (C) and (D) out of which ONE or MORE are correct.

The figure shows a snapshot of a vibrating string at $t = 0$. The particle P is observed moving up with a velocity $20\sqrt{3}$ cm/s. The tangent at P makes an angle 60° with x -axis. Then (μ mass per unit length of string = 50gm/m)



- A. the wave is moving along negative x -axis
- B. the wave is moving along positive x -axis
- C. the equation of wave is given by $y = 0.4 \sin 2\pi(5t - \frac{x}{4} + \frac{1}{12})$
- D. the total energy carried by the wave per cycle is 3.6×10^{-6} J

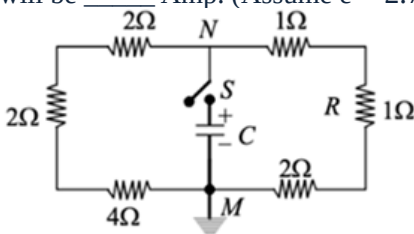
Physics Numerical (Maximum Marks: 18)

Question No. 1

Numerical Type

The answer has to be filled into the input box provided below.

A capacitor of capacity $6\mu\text{F}$ and initial charge $160\mu\text{C}$ is connected with a key S and resistance as shown in figure. Point M is earthed. If key is closed at $t = 0$ then the current through resistance $R = 1\Omega$ at $t = 16\mu\text{sec}$ will be ____ Amp. (Assume $e = 2.7$)

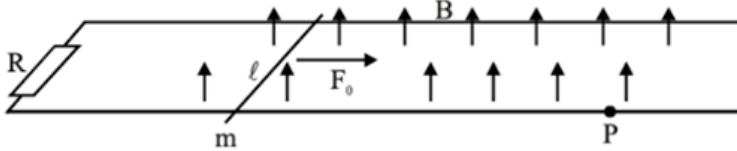


Question No. 2

Numerical Type

The answer has to be filled into the input box provided below.

The long, horizontal pair of rails shown in the figure is connected using resistance R . The distance between the rails is ℓ , the electrical resistance of the rails is negligible. A conducting wire of mass m and length ℓ can slide without friction on the pair of rails, in a vertical, homogeneous magnetic field of induction B . A force of magnitude F_0 is exerted for sufficiently long time onto the conducting wire, so that the speed of the wire becomes nearly constant. The force F_0 is now removed at a certain point P . The distance the conducting wire cover on rails from point P before stopping is $(316 + n)$ meters. Find 'n'. (Given : $F_0 = 20$ N, $m = 1.6$ gm , $R = 0.01\Omega$, $\ell = 10$ cm, $B = 0.1$ T)



Question No. 3

Numerical Type

The answer has to be filled into the input box provided below.

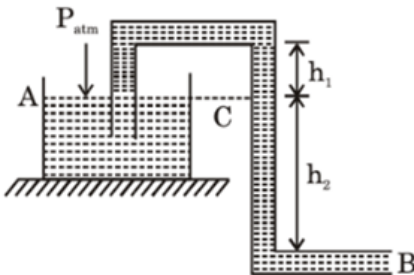
A body of mass $m = 4$ kg starts moving with velocity v_0 in a straight line in such a way that on the body work is being done at the rate which is proportional to the square of velocity as given by $P = \beta v^2$ where $\beta = \frac{0.693}{2}$. Find the time elapsed in seconds before velocity of body is doubled (nearest integer).

Question No. 4

Numerical Type

The answer has to be filled into the input box provided below.

A tube of uniform cross-section is used to siphon water from a vessel, as shown in the Figure. Assuming that the cross-section of the vessel is very much greater than that of the tube, find the maximum value of height h_1 (in m), for which the siphon will work. [Take $h_2 = 3$ m, atmospheric pressure $P_{\text{atm}} = 10^5 \text{ Nm}^{-2}$, density of water $\rho = 1000 \text{ kg m}^{-3}$ and acceleration due to gravity $g = 10 \text{ ms}^{-2}$]



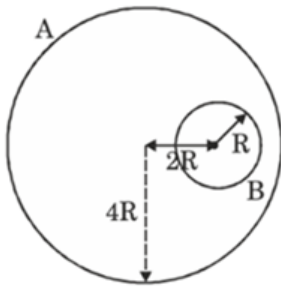
Question No. 5

Numerical Type

The answer has to be filled into the input box provided below.

Figure shows a uniformly charged spherical shell B of charge ($q_B = q$) kept inside uniformly charged spherical shell A of charge ($q_A = q$). Let electric field due to A and B at any point are \vec{E}_A and \vec{E}_B respectively. If dV

represents elementary volume, then the value of $\frac{\int_{\text{entire space}} \vec{E}_B \cdot \vec{E}_B dV}{\int_{\text{entire space}} \vec{E}_A \cdot \vec{E}_B dV}$ will be

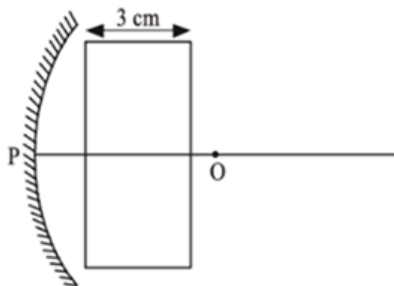


Question No. 6

Numerical Type

The answer has to be filled into the input box provided below.

An object is placed 21 cm in front of a concave mirror of radius of curvature 10 cm. A glass slab of thickness 3 cm and refractive index 1.5 is then placed close to the mirror in the space between the object and the mirror. Find the position of the final image (in cm) formed from mirror.



Physics Paragraph Type (Maximum Marks: 12)

Question No. 1

Only One Option Correct Type

Each question has four choices (A), (B), (C) and (D) out of which ONLY ONE is correct.

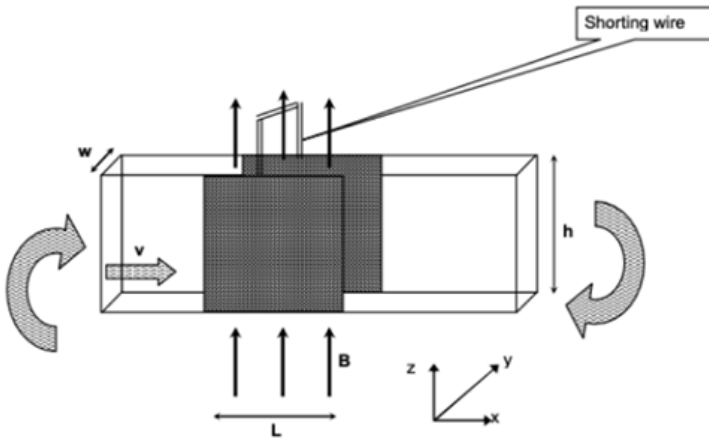
Paragraph

The production of electrical power through the use of a conducting fluid moving through a magnetic field is referred to as a MAGNETO HYDRODYNAMIC (MHD) GENERATOR. A simplistic way of functioning of MHD is shown in the figure. In the figure shown, a horizontal rectangular plastic pipe of width w and height h is filled with mercury of electrical conductivity σ . A pressure difference ΔP is produced by a turbine which drives this fluid with a constant speed V_0 . The two opposite vertical walls of a section of the pipe with length L are made of copper.

To simplify the situation we assume the following.

- Although the fluid is viscous, its speed is uniform over the entire cross-section.
- The speed of the fluid is always proportional to the net external force acting upon it.
- The fluid is incompressible.

These conducting walls are electrically shorted with an external wire as shown in the figure. The steady state velocity of the fluid after application of magnetic field is v .



Question

The current flowing in the shorting wire in the steady state is

- A. $BLh\sigma v$
- B. $BL\omega\sigma v$
- C. $\frac{BL^2h\sigma v}{\omega}$
- D. $Bvh\sigma\omega$

Question No. 2

Only One Option Correct Type

Each question has four choices (A), (B), (C) and (D) out of which ONLY ONE is correct.

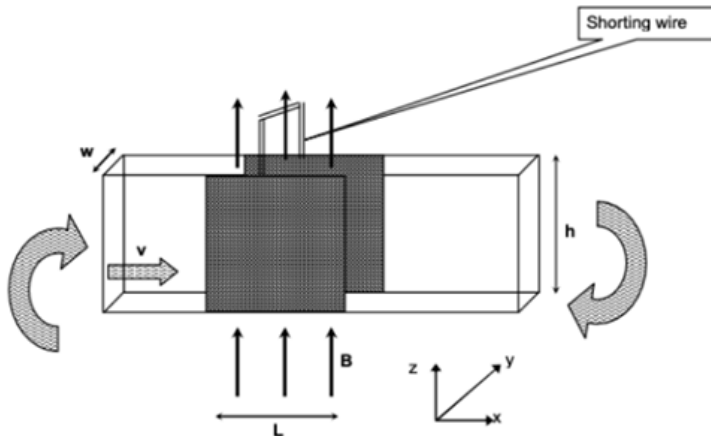
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Question

Taking the assumption taken in the MHD working that speed of the fluid is always proportional to the net external force acting upon it, the expression of steady state velocity v after application of magnetic field is

- $\frac{\Delta P}{B^2 L \sigma}$
- $\frac{\Delta P v_0}{\Delta P + B^2 L \sigma v_0}$
- $\frac{\Delta P h v_0}{\Delta P h + B^2 L w \sigma v_0}$
- $\frac{\Delta P v_0}{\Delta P + B^2 w v_0}$

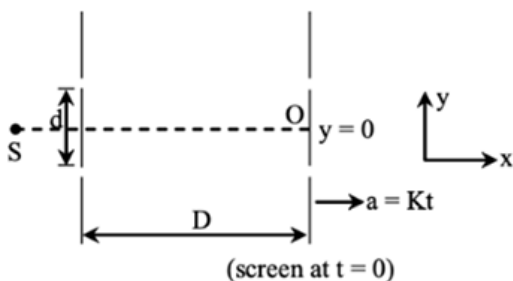
Question No. 3

Only One Option Correct Type

Each question has four choices (A), (B), (C) and (D) out of which ONLY ONE is correct.

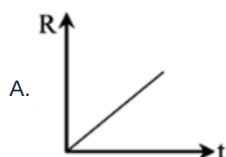
Paragraph

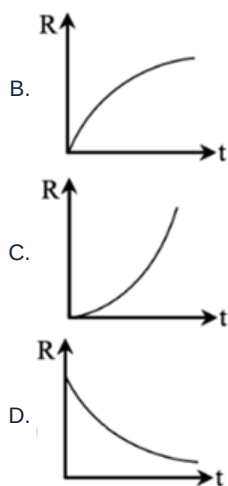
Consider YDSE arrangement shown. The screen in the arrangement starts accelerating from rest in the positive x -direction with acceleration $a = Kt$ at time $t = 0$ (here K is a constant and t is time)



Question

Rate of change of fringe width (R) for the fringe pattern on the screen varies with time as





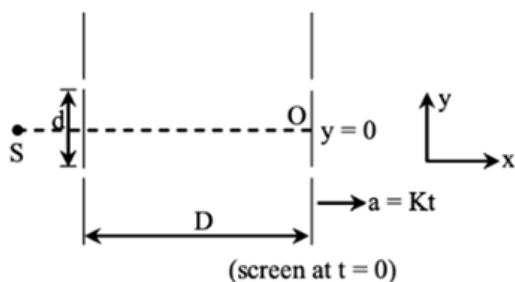
Question No. 4

Only One Option Correct Type

Each question has four choices (A), (B), (C) and (D) out of which ONLY ONE is correct.

Paragraph

Consider YDSE arrangement shown. The screen in the arrangement starts accelerating from rest in the positive x-direction with acceleration $a = Kt$ at time $t = 0$ (here K is a constant and t is time)



Question

Velocity of n^{th} maxima (above "O") at any time t in ground frame is -

- A. $\frac{Kt^2}{2} (\hat{i} + \frac{n\lambda}{d} \hat{j})$
- B. $Kt(\hat{i} + \frac{n\lambda}{d} \hat{j})$
- C. $\frac{Kt^2}{2} (\frac{n\lambda}{d} \hat{i} + \hat{j})$
- D. None of these

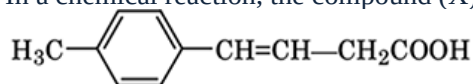
Chemistry Multiple Correct (Maximum Marks: 32)

Question No. 1

One or More Options Correct Type

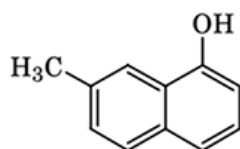
The question has four choices (A), (B), (C) and (D) out of which ONE or MORE are correct.

In a chemical reaction, the compound (X)



4(p-tolyl)-3 butenoic acid

Can rather conveniently be converted to another compound (Y) which is



7-methyl 1-naphthol

Which of the following is/are suitable for this purpose.

- A. Trans isomer of (X) can be treated with anhydrous AlCl_3 .
- B. Cis isomer of (X) can be treated with HF.
- C. Trans isomer of (X) can be treated with conc. H_3PO_4 .
- D. Cis isomer of (X) can be treated with NaOH.

Question No. 2

One or More Options Correct Type

The question has four choices (A), (B), (C) and (D) out of which ONE or MORE are correct.

Which of the following statements is/are false ?

- A. Chromium on reaction with Concentrated Nitric Acid liberates Nitrogen Dioxide gas.
- B. In solid state, PCl_5 exists as $[\text{PCl}_4]^- [\text{PCl}_6]^+$
- C. $\text{H}_2\text{P}_2\text{O}_6$ is formed when Phosphorus Acid reacts with Bromine.
- D. The Hydrolysis of XeF_2 , XeF_4 , XeF_6 all yield Oxygen Gas.

Question No. 3

One or More Options Correct Type

The question has four choices (A), (B), (C) and (D) out of which ONE or MORE are correct.

For real gases various equation of state are known, like Vanderwaals equation, Berthelot's equation, virial equation, Dieterici's equation etc. Although Vander waal's equation is easy to use, but the experimental value of 'Z' (compressibility factor) at critical condition does not match with the predicted value [Experimental ≈ 0.27 , predicted by Vanderwaal ≈ 0.375]. The experimental value more appropriately match by Dieterici's equation :

$$P = \left(\frac{RT}{V_m - b} \right) \cdot e^{-\frac{a}{V_m RT}}$$

Where a, b are Dieterici's constant, V_m = molar volume, P = Pressure & T = temperature. P_c , V_c & T_c in terms of Dieterici's constant will be respectively -

- A. $\frac{a}{4b^2e^2}$, $2b$, $\frac{a}{4bR}$
- B. $\frac{a}{27b^2}$, $3b$, $\frac{8a}{27Rb}$
- C. $\frac{a}{4b^2e^2}$, $3b$, $\frac{a}{4bR}$
- D. $\frac{a}{b^2e^2}$, $2b$, $\frac{a}{4bR}$

Question No. 4

One or More Options Correct Type

The question has four choices (A), (B), (C) and (D) out of which ONE or MORE are correct.

Which of the following statements is/are false ?

- A. In $[\text{PtCl}_2(\text{NH}_3)_4]^{2+}$ complex ion, the cis-form is optically active, while trans-form is optically inactive
- B. In $[\text{Fe}(\text{C}_2\text{O}_4)_3]^{3-}$, geometrical isomerism does not exist, while optical isomerism exists
- C. $[\text{Mabcd}]^{n\pm}$ square planar complexes exhibit both optical as well as geometrical isomerism
- D. In $[\text{Mabcd}]^{n\pm}$ tetrahedral complexes, optical isomerism cannot be observed

Question No. 5

One or More Options Correct Type

The question has four choices (A), (B), (C) and (D) out of which ONE or MORE are correct.

In order to study effect of temperature on rate of reaction, another term " σ " is defined as $\sigma = \frac{1}{k} \frac{dk}{dT}$ [where k is rate constant and T represents temperature]. If for a reaction, the value of $\sigma = \frac{1.25 \times 10^6}{T^3} \text{ K}^{-1}$ [T should be in Kelvin], then identify the correct statement(s).

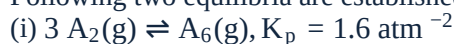
- A. Activation energy for the given reaction at 250 Kelvin is 10 Kcal/mol.
- B. Activation energy for the given reaction at 2000 Kelvin is 1.25 Kcal/mol.
- C. Activation energy of this reaction increases on increasing temperature.
- D. The value of $\frac{dk}{dT}$ at 2000 Kelvin is $0.625 \times k$.

Question No. 6

One or More Options Correct Type

The question has four choices (A), (B), (C) and (D) out of which ONE or MORE are correct.

Following two equilibria are established on mixing two gases A_2 and C .



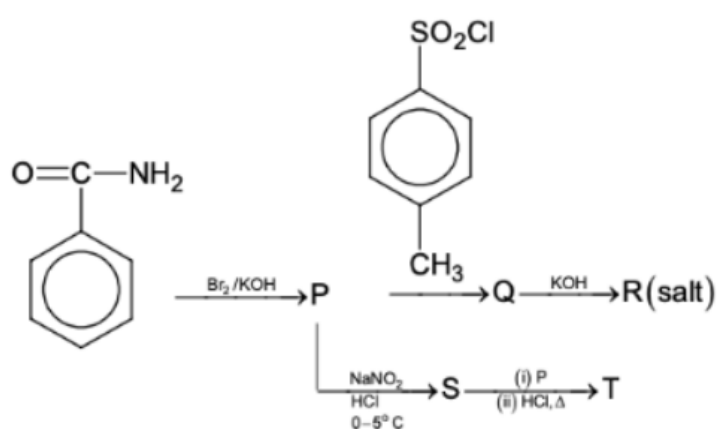
A_2 and C are mixed in 2 : 1 molar ratio in a sealed container at a constant temperature. If the total pressure of the reaction mixture is given to be 1.4 atm and partial pressure of A_6 to be 0.2 atm at equilibrium, then identify the correct statement(s) of the following:

- A. Equilibrium partial pressure of $A_2(g)$ is 0.5 atm .
- B. Equilibrium partial pressure of ' C ' is 0.4 atm
- C. Equilibrium partial pressure of $A_2C(g)$ is 0.3 atm
- D. K_p of reaction (ii) is 1.5 atm^{-1}

Question No. 7

One or More Options Correct Type

The question has four choices (A), (B), (C) and (D) out of which ONE or MORE are correct.



Which of the following is(are) correct?

- A. $P =$
- B. R is
- C. S is
- D. T is

Question No. 8

One or More Options Correct Type

The question has four choices (A), (B), (C) and (D) out of which ONE or MORE are correct.

Which is / are correct statement?

- $W_{\text{on the system, adiabatic}} > W_{\text{on the system, isothermally}}$ in an ideal gas compression from same initial state to
- same final volume
 - The value of $\gamma (\gamma = \frac{C_p}{C_v})$ remains constant for diatomic gas at all temperature
 - Entropy increases when an ideal gas expanded isothermally.
 - $\Delta_r H$ & $\Delta_r S$ both are +ve for the decomposition of $\text{MgCO}_3(\text{s})$.

Chemistry Numerical (Maximum Marks: 18)

Question No. 1

Numerical Type

The answer has to be filled into the input box provided below.

Note: Do not care about the Dimensions of the Physical Quantities in this Problem.

The energy of a particle in a 1 Dimensional Box (1D Box) is given by

$$E_n = \frac{n^2 h^2}{8mL^2}$$

Where,

n is an Integer starting from 1 and represents the quantum number for the energy state

$\frac{h^2}{8mL^2}$ is assumed to be a constant with a value equal to β .

h is the Planck's Constant

m is the Mass of the Particle

L is the Length of the Box.

Note that no more than 2 electrons can occupy an Orbital (Energy Level).

The energy difference between the HOMO and LUMO of a hypothetical particle having 47 Electrons is given by $f\beta$.

HOMO: Highest Occupied Molecular Orbital

LUMO: Lowest Unoccupied Molecular Orbital.

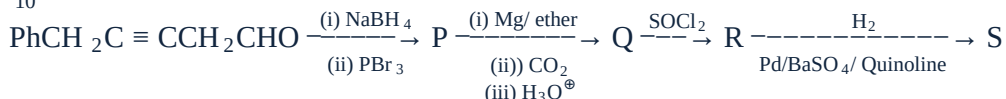
Then, the Value of f is _____.

Question No. 2

Numerical Type

The answer has to be filled into the input box provided below.

In the following sequence, product P, Q, R and S are formed, if the molar mass of S is M, then what is value of $\frac{M}{10}$.



Question No. 3

Numerical Type

The answer has to be filled into the input box provided below.

The total number of metals that can be the best refined by liquation method from following - Copper, bismuth, silver, lead, gold, zinc, mercury ?

Question No. 4

Numerical Type

The answer has to be filled into the input box provided below.

Henry's law constants for O_2 and N_2 dissolved in water at 273 K are 2.03×10^8 Pa and 5.07×10^8 Pa, respectively. A sample of water at a temperature just above 273 K was equilibrated with air (21% oxygen and 79% N_2) at 1 atm. The dissolved gas was separated from a sample of this water and then dried. Determine the composition of N_2 in percentage (nearest integer).

Question No. 5

Numerical Type

The answer has to be filled into the input box provided below.

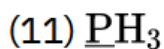
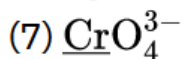
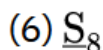
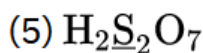
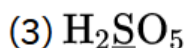
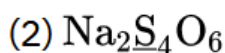
A compound (A) having molecular formula $C_7H_{11}Br$ is optically active. A reacts with HBr in the absence of peroxide to give isomeric products (B) and (C). Treating (A) with potassium-t-butoxide gives (D). (D) on reductive ozonolysis gives two moles of formaldehyde and one mole of 1,3 cyclopentane dione. (A) in the presence of peroxide reacts with HBr to give (E). The total number of chiral carbon in one molecule of (E) is:

Question No. 6

Numerical Type

The answer has to be filled into the input box provided below.

Identify the species in which the "actual" as well as the "average" Oxidation number of the marked element is/are an EVEN integer

**Chemistry Paragraph Type (Maximum Marks: 12)**

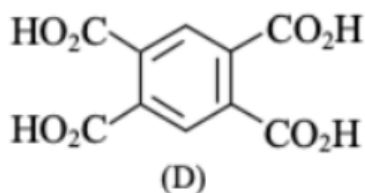
Question No. 1

Only One Option Correct Type

Each question has four choices (A), (B), (C) and (D) out of which ONLY ONE is correct.

Paragraph

Compound (A), $C_{11}H_{12}O$, which gave no Tollen's test, was treated with $LiAlH_4$, followed by dilute acid, to give compound (B), which could be resolved into enantiomers. When optically active (B) was treated with CrO_3 in pyridine, an optically inactive sample of (A) was obtained. Heating (A) with hydrazine in base gave hydrocarbon (C), which, when heated with alkaline $KMnO_4$ followed by acidification, gave carboxylic acid (D).



Another compound (X), $C_6H_{12}O_2$, was found to be optically active and was slowly oxidized to an optically active carboxylic acid (Y), $C_6H_{12}O_3$, by $[Ag(NH_3)_2]^+$. Oxidation of (X) by anhydrous CrO_3 gave an optically inactive compound that reacted with Zn amalgam /HCl to give 3-methylpentane. With aqueous H_2CrO_4 , compound (X) was oxidized to an optically inactive dicarboxylic acid (Z), $C_6H_{10}O_4$.

Question

The most probable structure of (A) will be

- A.
- B.
- C.
- D.

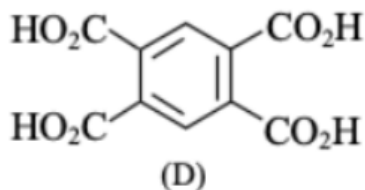
Question No. 2

Only One Option Correct Type

Each question has four choices (A), (B), (C) and (D) out of which ONLY ONE is correct.

Paragraph

Compound (A), $C_{11}H_{12}O$, which gave no Tollen's test, was treated with $LiAlH_4$, followed by dilute acid, to give compound (B), which could be resolved into enantiomers. When optically active (B) was treated with CrO_3 in pyridine, an optically inactive sample of (A) was obtained. Heating (A) with hydrazine in base gave hydrocarbon (C), which, when heated with alkaline $KMnO_4$ followed by acidification, gave carboxylic acid (D).



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Question

The structure of compound (X) would be

- A. $\text{OHCCH}_2-\overset{\text{CH}_3}{\underset{|}{\text{CH}}}-\text{CH}_2\text{CHO}$
- B. $\text{OHCCH}_2-\overset{\text{CH}_3}{\underset{|}{\text{CH}}}-\text{COCH}_3$
- C. $\text{HOH}_2\text{CCH}_2-\overset{\text{CH}_3}{\underset{|}{\text{CH}}}-\text{COCH}_3$
- D. $\text{OHCCH}_2-\overset{\text{CH}_3}{\underset{|}{\text{CH}}}-\text{CH}_2\text{CH}_2\text{OH}$

Question No. 3

Only One Option Correct Type

Each question has four choices (A), (B), (C) and (D) out of which ONLY ONE is correct.

Paragraph

There are 4 glass bottles containing water solutions. These bottles are not labeled. Bottles 1, 2, 3 Contained colourless solutions, while bottle 4 contained a blue solution. Each bottle is having any one solution among the following (Each bottle is having different solution).

Copper (III) sulphate

Sodium carbonates

Lead nitrate

Hydrochloric acid

By mixing samples of the contents of the bottles, in pairs, following observations are made:

- | | | |
|-----|---------------------|------------------------|
| (1) | Bottle 1 + Bottle 2 | White ppt. |
| (2) | Bottle 1 + Bottle 3 | White ppt. |
| (3) | Bottle 1 + Bottle 4 | White ppt. |
| (4) | Bottle 2 + Bottle 3 | Colourless gas evolved |
| (5) | Bottle 2 + Bottle 4 | No visible reaction |
| (6) | Bottle 3 + Bottle 4 | Blue ppt. |

Question

Colourless solution present in Bottle 1 is :

- A. CuSO_4
- B. HCl
- C. $\text{Pb}(\text{NO}_3)_2$
- D. Na_2CO_3

Question No. 4

Only One Option Correct Type

Each question has four choices (A), (B), (C) and (D) out of which ONLY ONE is correct.

Paragraph

There are 4 glass bottles containing water solutions. These bottles are not labeled. Bottles 1, 2, 3 Contained colourless solutions, while bottle 4 contained a blue solution. Each bottle is having any one solution among the following (Each bottle is having different solution).

Copper (III) sulphate

Sodium carbonates

Lead nitrate

Hydrochloric acid

By mixing samples of the contents of the bottles, in pairs, following observations are made:

- (1) Bottle 1 + Bottle 2 White ppt.
- (2) Bottle 1 + Bottle 3 White ppt.
- (3) Bottle 1 + Bottle 4 White ppt.
- (4) Bottle 2 + Bottle 3 Colourless gas evolved
- (5) Bottle 2 + Bottle 4 No visible reaction
- (6) Bottle 3 + Bottle 4 Blue ppt.

Question

Nature of gas evolved in observation (4) is:

- A. Acidic
- B. Neutral
- C. Basic
- D. Amphoteric

Mathematics Multiple Correct (Maximum Marks: 32)

Question No. 1

One or More Options Correct Type

The question has four choices (A), (B), (C) and (D) out of which ONE or MORE are correct.

Let $P = |z^5 + \bar{z}^3 + 6z| - 2|z^4 + 1|$ and $|z| = 1$ and Q is number of real roots of $5 - x = \sqrt{x + \sqrt{x + \sqrt{x}}}$, where $z \in \mathbb{C}$, $x \in \mathbb{R}$, then which of the following is/are FALSE ?
(where P_{\min} and P_{\max} are minimum and maximum value of P respectively)

- A. $P_{\min} + Q = 5$
- B. $P_{\max} + Q = 4$
- C. $P + Q \in [4, 5]$
- D. $P - Q \in [3, 4]$

Question No. 2

One or More Options Correct Type

The question has four choices (A), (B), (C) and (D) out of which ONE or MORE are correct.

If $\Psi_1 = \left(\frac{1}{n^6} + \frac{32}{n^6} + \frac{243}{n^6} + \dots + \frac{1}{n}\right)$ and $\Psi_2 = \left(\frac{1}{n^6} + \frac{32}{n^6} + \frac{243}{n^6} + \dots + \frac{(n-1)^5}{n^6}\right)$, then which of the following is/are true?

$[x]$ denotes greatest integer less than or equal to x

- A. $\lim_{n \rightarrow \infty} [6\Psi_1] = 1$
- B. $\lim_{n \rightarrow \infty} [3(\Psi_1 + \Psi_2)] = 1$
- C. $\lim_{n \rightarrow \infty} [3(\Psi_1 + \Psi_2)] = 0$
- D. $\lim_{n \rightarrow \infty} [6\Psi_2] = 0$

Question No. 3

One or More Options Correct Type

The question has four choices (A), (B), (C) and (D) out of which ONE or MORE are correct.

Let A be a square matrix of order 3 which satisfies the equation $A^4 - 8A^3 + 24A^2 - 37A + 16I = 0$ and $B = A - 2I$ be another matrix. If $\text{Det } A = 5$ and $\text{Det } B > 0$, A^{-1} represent inverse of matrix A , then

- A. $\text{Det}(\text{Adj}(I - 2A^{-1}))$ is equal to 1
- B. $\text{Det}(\text{Adj}(\frac{B}{5})^{-1})$ is equal to 625
- C. $\text{Det}(I + 2B^{-1})$ is equal to 1
- D. $\text{Det } B = 25$

Question No. 4

One or More Options Correct Type

The question has four choices (A), (B), (C) and (D) out of which ONE or MORE are correct.

Consider three vectors $\vec{v}_1 = \alpha\hat{i} + \beta\hat{j} + \gamma\hat{k}$, $\vec{v}_2 = \beta\hat{i} + \gamma\hat{j} + \alpha\hat{k}$, $\vec{v}_3 = \gamma\hat{i} + \alpha\hat{j} + \beta\hat{k}$, where α, β, γ are determined by randomly selecting 3 cards from a pack of 52 cards, such that a King represents 13, a Queen represents 12, a Jack represents 11 while an Ace represents 1 and remaining cards represent number on them, then

- A. probability that $\vec{v}_1, \vec{v}_2, \vec{v}_3$ are linearly independent is $\frac{1}{425}$.
- B. probability that $\vec{v}_1, \vec{v}_2, \vec{v}_3$ are linearly dependent is $\frac{1}{425}$.
- C. probability that volume of tetrahedron formed by $\vec{v}_1, \vec{v}_2, \vec{v}_3$ is maximum is $\frac{24}{52C_3}$
- D. probability that volume of parallelepiped formed by $\vec{v}_1, \vec{v}_2, \vec{v}_3$ is minimum is $\frac{13 \cdot 4C_3}{52C_3}$.

Question No. 5

One or More Options Correct Type

The question has four choices (A), (B), (C) and (D) out of which ONE or MORE are correct.

$f : [0, 1] \rightarrow \mathbb{R}$ is a function such that $f'(x)$ is continuous and $\int_0^1 f(x)dx = 0$, if maximum value of $|f'(x)|$ when $x \in [0, 1]$ is 24, then possible value of $\left| \int_0^x f(t)dt \right|$, $x \in [0, 1]$, is/are:

- A. 1
- B. 2
- C. 3
- D. 4

Question No. 6

One or More Options Correct Type

The question has four choices (A), (B), (C) and (D) out of which ONE or MORE are correct.

Let $f : \mathbb{R} \rightarrow \mathbb{R}$ be defined as

$$f(x) = \begin{cases} \lim_{n \rightarrow \infty} \left(\frac{[x]}{1+n^2} + \frac{3[x]}{2+n^2} + \frac{5[x]}{3+n^2} + \dots + \frac{(2n-1)[x]}{n+n^2} \right), & x \neq \frac{\pi}{2} \\ 1, & x = \frac{\pi}{2} \end{cases}, \text{ where } [.] \text{ denotes GIF, then}$$

which of the following statement(s) is (are) correct?

- A. $f(x)$ is injective but not surjective
- B. $f(x)$ is non-differentiable at $x = \frac{\pi}{2}$
- C. $f(x)$ is discontinuous at all integers and continuous at $x = \frac{\pi}{2}$
- D. $f(x)$ is unbounded function.

Question No. 7

One or More Options Correct Type

The question has four choices (A), (B), (C) and (D) out of which ONE or MORE are correct.

The curve satisfying the differential equation $(2x^2y - 2y^4)dx + (2x^3 + 3xy^3)dy = 0$ and passing through $(1, 1)$ is given by $2\ell n(xy) + \frac{y^m}{x^n} = 1, m, n \in \mathbb{N}$, then

- A. $m + n = 5$
- B. $m + n = 4$
- C. $m \times n = 6$
- D. $m \times n = 4$

Question No. 8

One or More Options Correct Type

The question has four choices (A), (B), (C) and (D) out of which ONE or MORE are correct.

Tangent is drawn at any point (x_1, y_1) other than vertex on the parabola $y^2 = 4ax$. If tangents are drawn from any point on this tangent to the circle $x^2 + y^2 = a^2$ such that all the chords of contact pass through a fixed point (x_2, y_2) then

- A. x_1, a, x_2 are in G.P.
- B. $\frac{y_1}{2}, a, y_2$ are in G.P.
- C. $-4, \frac{y_1}{y_2}, \frac{x_1}{x_2}$ are in G.P.
- D. $x_1 x_2 + y_1 y_2 = a^2$.

Mathematics Numerical (Maximum Marks: 18)

Question No. 1

Numerical Type

The answer has to be filled into the input box provided below.

The polynomial $P(x) = (1 + x + x^2 + \dots + x^{17})^2 - x^{17}$ has 34 complex roots of the form $z_k = r_k [\cos(2\pi a_k) + i \sin(2\pi a_k)]$, $k = 1, 2, 3, \dots, 34$ with $0 < a_1 \leq a_2 \leq a_3 \leq \dots \leq a_{34} < 1$ and $r_k > 0$. Given that $a_1 + a_2 + a_3 + a_4 + a_5 = \frac{p}{q}$, where p, q are relatively prime positive integers, then $(p + q)$ is equal to

Question No. 2

Numerical Type

The answer has to be filled into the input box provided below.

Number of ways in which two distinct natural numbers can be selected out of first 100 natural numbers so that sum of their cubes is multiple of 8 is N, then $[\frac{N}{200}]$ is equal to (where $[.]$ denotes greatest integer function)

Question No. 3

Numerical Type

The answer has to be filled into the input box provided below.

Consider a tetrahedron Die that has four integers 1, 2, 3 and 4 written on its faces. Roll the Die 2000 times and for each i , $1 \leq i \leq 4$ let $f(i)$ represent the number of times i is written on the bottom face.

Let A denote total sum of the numbers on the bottom face for these 2000 rolls, if $A^4 = 6144f(1)f(2)f(3)f(4)$, then

The value of $\left| \frac{f(1)-f(2)}{f(4)} \right|$ is equal to

Question No. 4

Numerical Type

The answer has to be filled into the input box provided below.

Let

$$f(x) = ax^{17} + b \sin x \cdot \sin 2x \cdot \sin 3x + cx^2 \operatorname{sgn}(\sin x) + d \log(x + \sqrt{1+x^2}) + x(|x+1| - |x-1|) \left(\frac{e^x - e^{-x}}{e^x + e^{-x}} \right)$$

be defined on the set of real numbers, $(a > 0, b, c, d \in \mathbb{R})$ if $f(-7) = 7, f(-5) = -5, f(-2) = 3$, then the minimum number of zeroes of the equation $f(x) = 0$ is equal to _____.

Question No. 5

Numerical Type

The answer has to be filled into the input box provided below.

The plane $ax + by + cz = 1$ containing the line $\frac{x-3}{2} = \frac{y-1}{4} = \frac{z-2}{5}$ is rotated through an angle $\frac{\pi}{2}$ about this line to contain the point $(4, 3, 7)$. Value of $(2a + 3b + \frac{c}{8})$ equals

Question No. 6

Numerical Type

The answer has to be filled into the input box provided below.

If A be the area bounded by the curves $|y| = e^{-|x|} - \frac{1}{2}$ and $\frac{|x|+|y|}{2} + \left| \frac{|x|-|y|}{2} \right| \leq 2$, then $\frac{A-\ln 4}{2}$ equal to

Mathematics Paragraph Type (Maximum Marks: 12)

Question No. 1

Only One Option Correct Type

Each question has four choices (A), (B), (C) and (D) out of which ONLY ONE is correct.

Paragraph

Continuous probability distributions, a continuous distribution is one in which the variate may take any value between certain limits a and $b, a < b$. Suppose that the probability of the variate X falling in the infinitesimal interval $x - \frac{1}{2} dx$ to $x + \frac{1}{2} dx$ is expressible as $f(x)dx$, where $f(x)$ is a continuous function of x . Symbolically,

$$P\left(x - \frac{1}{2} dx \leq X \leq x + \frac{1}{2} dx\right) = f(x)dx \quad \text{where } f(x) \text{ is called the probability density function (abbreviated}$$

as p.d.f.) or simply density function. The continuous curve $y = f(x)$ is called probability curve; and when this is symmetrical, the distribution is said to be symmetrical. Clearly, the probability density function possesses the following properties

(i) $f(x) \geq 0$ for every x in the interval $[a, b], a < b$

(ii) $\int_a^b f(x)dx = 1, a, b > 0$ since the total area under the curve is unity

(iii) Furthermore, we define for any $[c, d]$, where $c, d \in [a, b], c < d$;

$$P(c \leq X \leq d) = \int_c^d f(x)dx \quad \dots(1)$$

We define $F(x)$, the cumulative distribution function (abbreviated as c.d.f.) of the random variate X where

$$F(x) = P(X \leq x)$$

$$\text{or } F(x) = \int_a^x f(x)dx \quad \dots(2)$$

Question

A random variable x has the density function $f(x) = \frac{c}{x^2+1}, -\infty < x < \infty$.

The probability that x^2 lies between $\frac{1}{3}$ and 1 is

- A. $\frac{1}{6}$
- B. $\frac{1}{3}$
- C. $\frac{2}{3}$
- D. $\frac{5}{6}$

Question No. 2

Only One Option Correct Type

Each question has four choices (A), (B), (C) and (D) out of which ONLY ONE is correct.

Paragraph

Continuous probability distributions, a continuous distribution is one in which the variate may take any value between certain limits a and b , $a < b$. Suppose that the probability of the variate X falling in the infinitesimal interval $x - \frac{1}{2} dx$ to $x + \frac{1}{2} dx$ is expressible as $f(x)dx$, where $f(x)$ is a continuous function of x . Symbolically,

$P(x - \frac{1}{2} dx \leq X \leq x + \frac{1}{2} dx) = f(x)dx$ where $f(x)$ is called the probability density function (abbreviated as p.d.f.) or simply density function. The continuous curve $y = f(x)$ is called probability curve; and when this is symmetrical, the distribution is said to be symmetrical. Clearly, the probability density function possesses the following properties

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$$F(x) = P(X \leq x)$$

$$\text{or } F(x) = \int_a^x f(x)dx \quad \dots(2)$$

Question

Suppose the life in hours (x) of a certain kind of radio tube has the probability density function $f(x) = \frac{100}{x^2}$ when $x > 100$ and zero when $x < 100$. Then the probability that none of three such tubes in a given radio set will have to be replaced during the first 150 hours of operation, is

- A. $\frac{1}{27}$
- B. $\frac{8}{27}$
- C. $\frac{1}{225}$
- D. $\frac{26}{27}$

Question No. 3

Only One Option Correct Type

Each question has four choices (A), (B), (C) and (D) out of which ONLY ONE is correct.

Paragraph

A sequence of ellipse E_1, E_2, \dots, E_n are constructed as follows: Ellipse E_n is drawn so as to touch ellipse E_{n-1} at the extremities of the major axis of E_{n-1} and to have its foci at the extremities of the minor axis of E_{n-1} .

Question

If eccentricity of E_n is independent of n , then eccentricity of the ellipse E_{n-2} is

- A. $\frac{3-\sqrt{5}}{2}$
- B. $\frac{\sqrt{5}-1}{2}$
- C. $\frac{2-\sqrt{3}}{2}$
- D. $\frac{\sqrt{3}-1}{2}$

Question No. 4

Only One Option Correct Type

Each question has four choices (A), (B), (C) and (D) out of which ONLY ONE is correct.

Paragraph

A sequence of ellipse E_1, E_2, \dots, E_n are constructed as follows: Ellipse E_n is drawn so as to touch ellipse E_{n-1} at the extremities of the major axis of E_{n-1} and to have its foci at the extremities of the minor axis of E_{n-1} .

Question

If eccentricity of ellipse E_n is independent of ' n ' then the locus of the mid point of chords of slope -1 of E_n (If the axis of E_n is along Y -axis)

- A. $(\sqrt{5}-1)x = 2y$
- B. $(\sqrt{5}+1)x = 2y$
- C. $(3-\sqrt{5})x = 2y$
- D. $(3+\sqrt{5})x = 2y$

